# STAT6061/STAT5008 – Causal Inference

# Part 4-1. Front-door Criterion

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# Front-door vs. Back-door Criteria

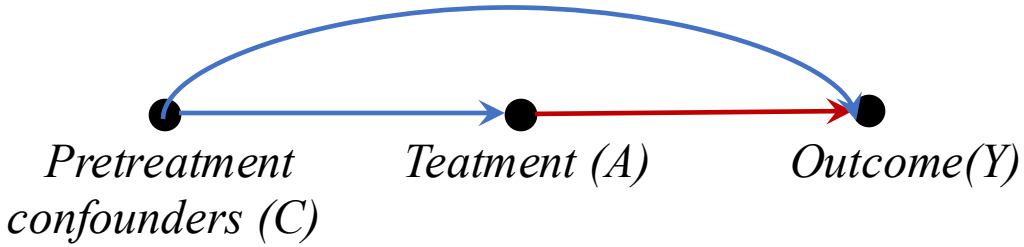
(Pearl, 1995)

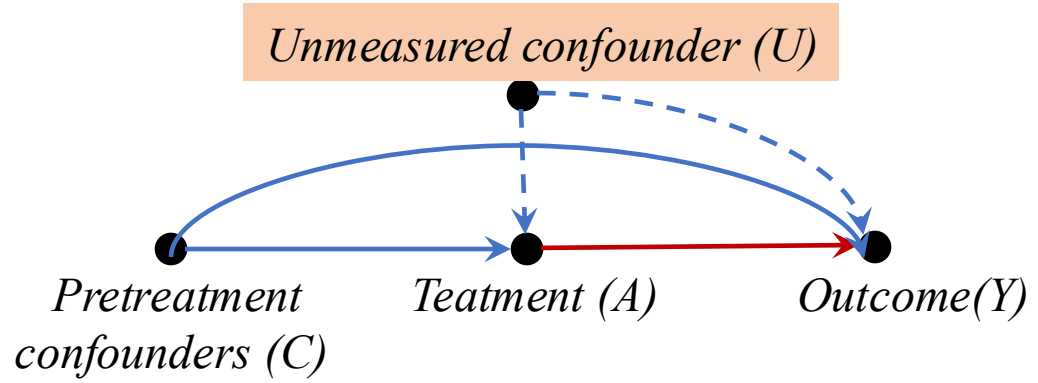
- The *back-door* and *front-door* criteria are two foundational tools introduced by Judea Pearl (1995) for identifying causal effects from observational data.

**Definition (Back-door criterion)**

*Blocks all non-causal (back-door) paths from treatment A to outcome Y by adjusting for observed confounders, isolating the direct causal effect.*

- The back-door criterion requires adjustment for confounding variables; that is, it is essentially equivalent to requiring that the assumption of exchangeability holds.
- However, the back-door criterion cannot be applied when confounding variables are not observed or recorded.

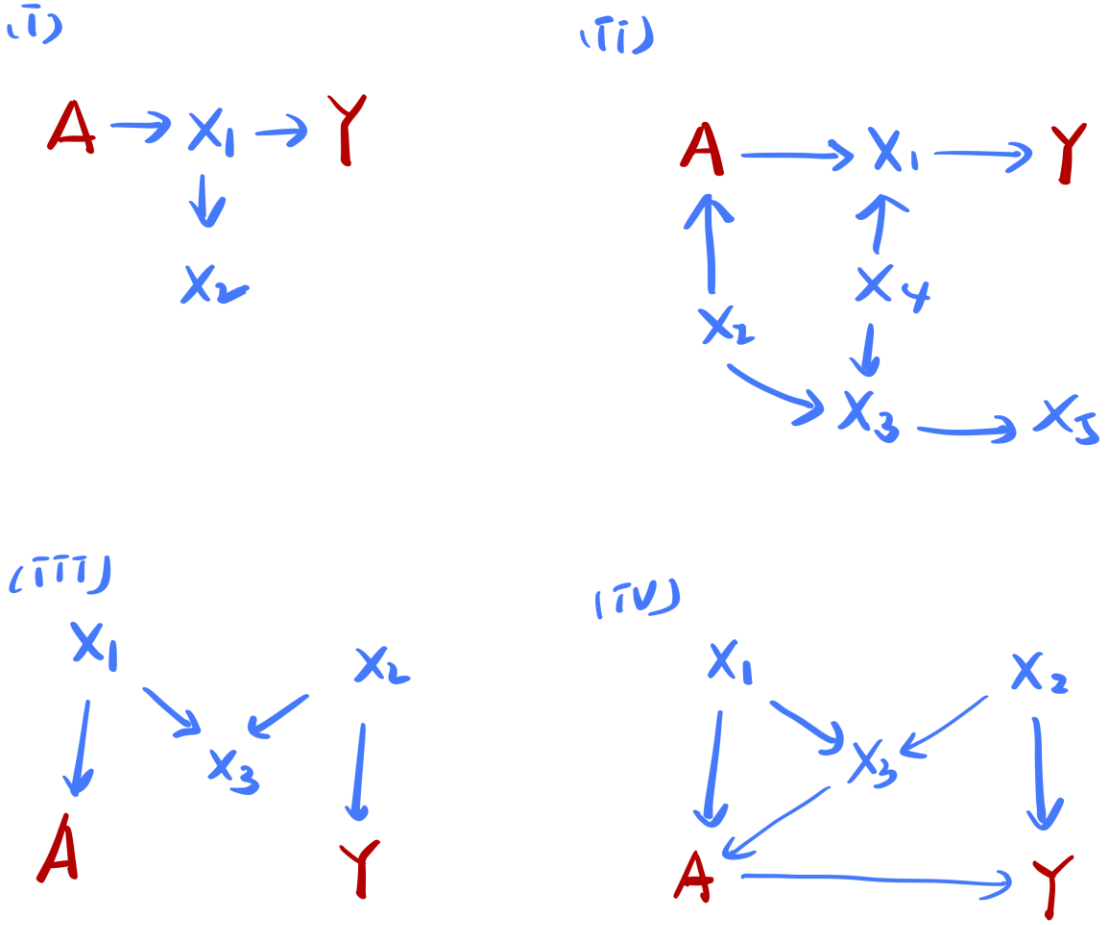


# Back-door Criteria

(Pearl and Mackenzie, 2018)

- Which variables should be adjusted for?



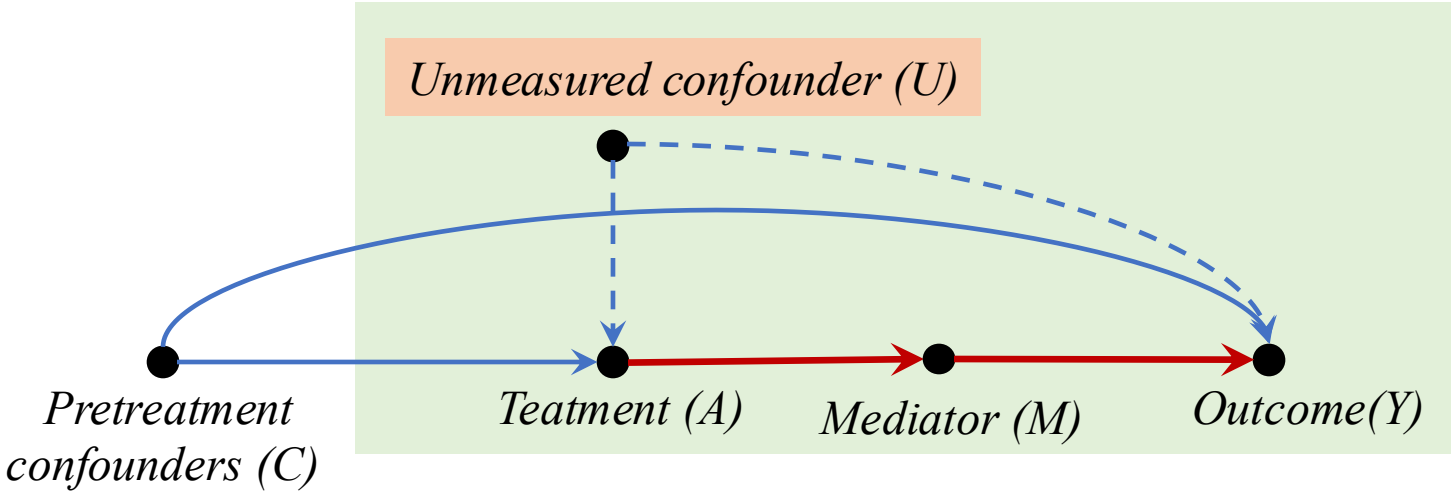
# Front-door vs. Back-door Criteria (cont.)

(Pearl, 1995)

- The *back-door* and *front-door* criteria are two foundational tools introduced by Judea Pearl (1995) for identifying causal effects from observational data.

**Definition (Front-door criterion)**

*Uses a mediator M to identify the causal effect of A on Y, even when confounding exists, by decomposing the causal pathway into identifiable components.*



# Identification: front-door criterion

- Omit the measurable confounder $C$ from the following theorem.

**Theorem 4.1**

*Suppose a set of variables $M$ satisfies the following conditions relative to an ordered pair of variables $(A, Y)$:*

*(Assumption i) $M$ intercepts all directed paths from $A$ to $Y$ (complete mediation),*
*(Assumption ii) there is no back-door path between $A$ and $M$, and*
*(Assumption iii) every back-door path between $M$ and $Y$ is blocked by $A$.*

*Then $\mathbb{E}(Y(a))$ is identifiable and is given by*

$$\int \{\sum \mathbb{E}(Y|A = a^*, M = m)\, Pr(A = a^*)\} \times Pr(M = m|A = a)\; dm$$

# Identification: front-door criterion (Assumptions)

- Conditions formulated through counterfactual notation

(Assumption 1): Complete Mediation:

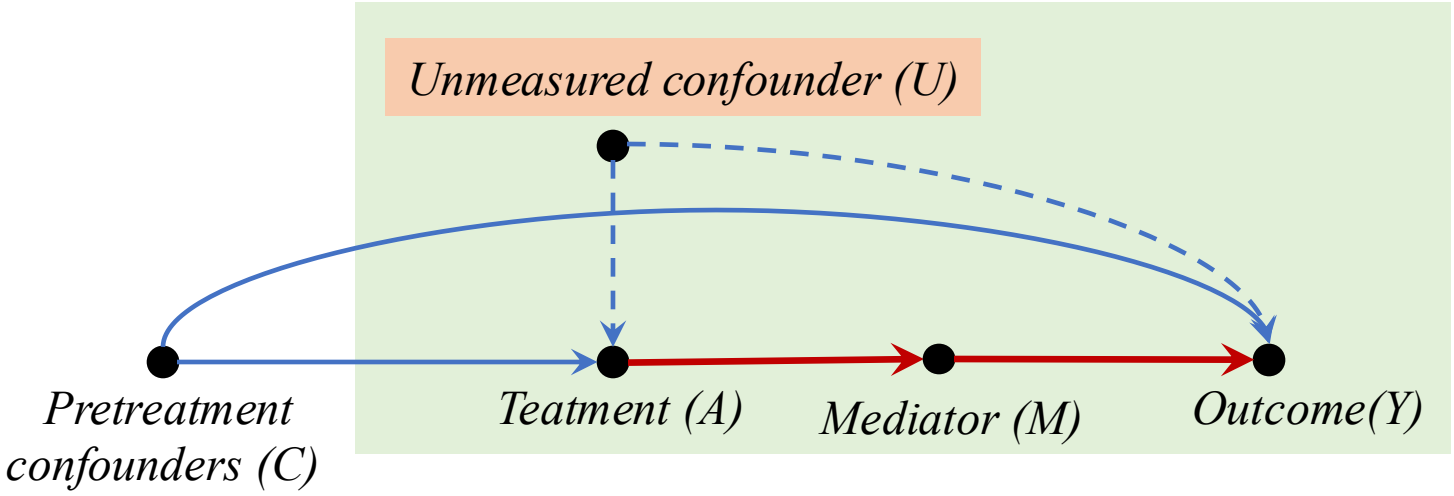
$$Y(a,m) = Y(m)$$

(Assumption 2): No unmeasured confounding between $A$ and $M$:

$$M(a) \perp A$$

(Assumption 3): All back-door paths from $M$ to $Y$ are blocked by $A$:

$$Y(m) \perp M|A \text{ and } Y(a,m) \perp M(a),$$



# Identification: front-door criteria (Proof)

(Consistent assumption)

$$\mathbb{E}\big(Y(a)\big) = \mathbb{E}(Y(a, M(a)))$$

$$= \int \mathbb{E}\big(Y\big(a, M(a)\big)\big|M(a) = m\big) \Pr(M(a) = m)\, dm = \int \mathbb{E}(Y(a, m)|M(a) = m) \Pr(M(a) = m)\, dm$$

(Consistent assumption)

**Part 1**

$$\mathbb{E}(Y(a, m)|M(a) = m) = \mathbb{E}\big(Y(a, m)\big) = \mathbb{E}\big(Y(m)\big) = \sum \mathbb{E}(Y(m)|A = a^*) \Pr(A = a^*)$$

(Assumption 3) (Assumption 1)

$$= \sum \mathbb{E}(Y(m)|A = a^*, M = m) \Pr(A = a^*)$$ (Assumption 3)

$$= \sum \mathbb{E}(Y|A = a^*, M = m) \Pr(A = a^*)$$ (Consistent assumption)

**Part 2**

$$\Pr(M(a) = m) = \Pr(M(a) = m|A = a) = \Pr(M = m|A = a)$$

(Assumption 2) (Consistent assumption)

Therefore, under the front-door criterion, $\mathbb{E}\big(Y(a)\big)$ is identified as

$$\int \{\sum \mathbb{E}(Y|A = a^*, M = m)\, Pr(A = a^*)\} \times Pr(M = m|A = a)\ dm$$

# Example (Inoue, Ritz, and Arah, 2022)

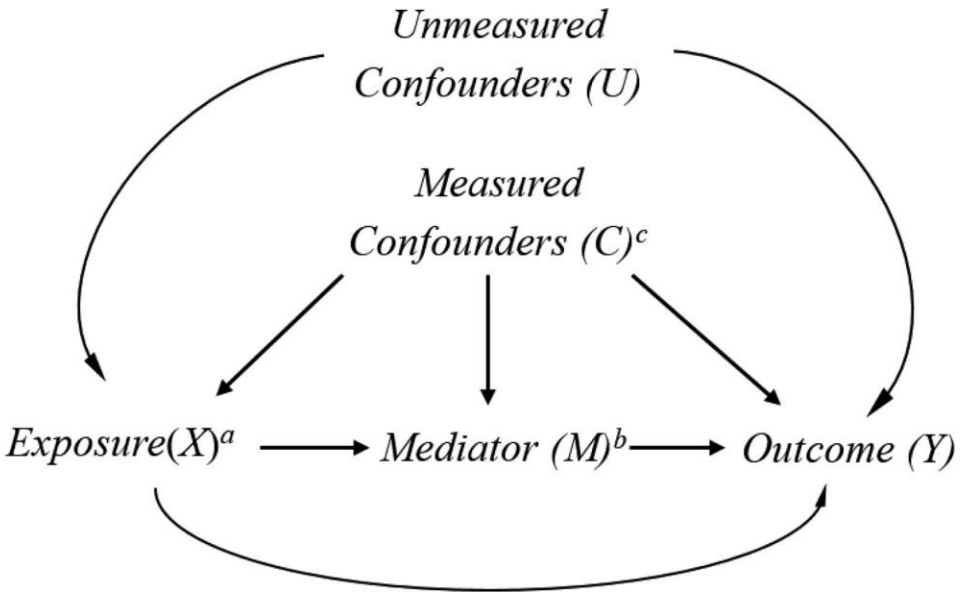
- Chronic pain is a major global health issue and a driver of opioid prescriptions.
- Observational studies on pain–opioid–mortality pathways are often confounded by unmeasured variables.

**Causal diagram illustrating the plausible relationships among chronic pain, opioid prescriptions, and mortality, accounting for both measured and unmeasured confounders.**

a. Chronic pain (*X*) was self-reported and defined as experiencing pain lasting at least three months.

b. Opioid use (*M*)

c. Measured covariates (*C*) include age, sex, race, educational attainment, poverty-income ratio, health insurance status, marital status, smoking, alcohol intake, and antidepressant use.




- Conclusion

Chronic pain was associated with a modest but significant increase in mortality risk through opioid use, with indirect effects observed at 3 years (OR = 1.06, 95% CI: 1.01–1.11) and 5 years (OR = 1.03, 95% CI: 1.01–1.06).

# Advantages and limitations

- **Advantages:**

1. Handles unmeasured confounding
2. Leverages observable mediators
3. Mechanism insight
4. Applicable in complex structures

- **Limitations:**

1. Rarely satisfied assumptions
2. Sensitivity to model misspecification

- Due to its stringent assumptions and the difficulty in finding appropriate mediators, the front-door criterion is often considered more of a theoretical tool than a practical solution.

# References

Inoue, K., Ritz, B., & Arah, O. A. (2022). Causal effect of chronic pain on mortality through opioid prescriptions: Application of the front-door formula. *Epidemiology*, *33*(4), 572-580.

Pearl, J. (1995). Causal Diagrams for Empirical Research. *Biometrika*, 82, 669–710.

Pearl, J., & Mackenzie, D. (2018). *The book of why: the new science of cause and effect*. Basic books.