STAT6061/STAT5008 – Causal Inference

Part 3-1*. Causal mediation analysis

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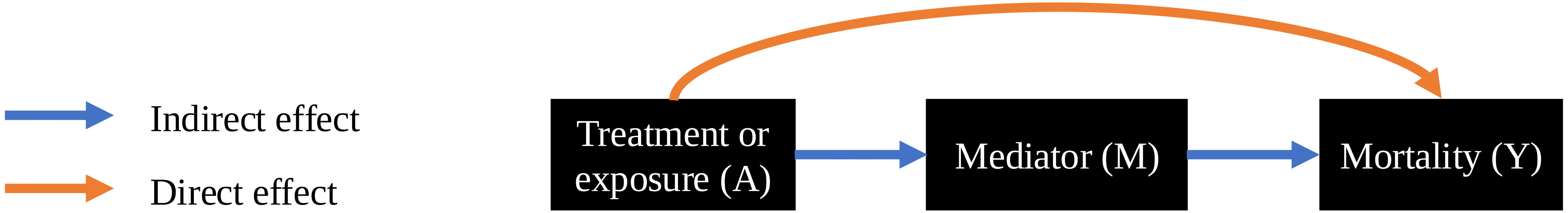
Mediation analysis

Effect decomposition of the total effect in the presence of a mediator



Prescriptive interpretation (Controlled direct and indirect effects)

- It is the effect of A after prescribing or intervening on the mediator M.

- Policy-making

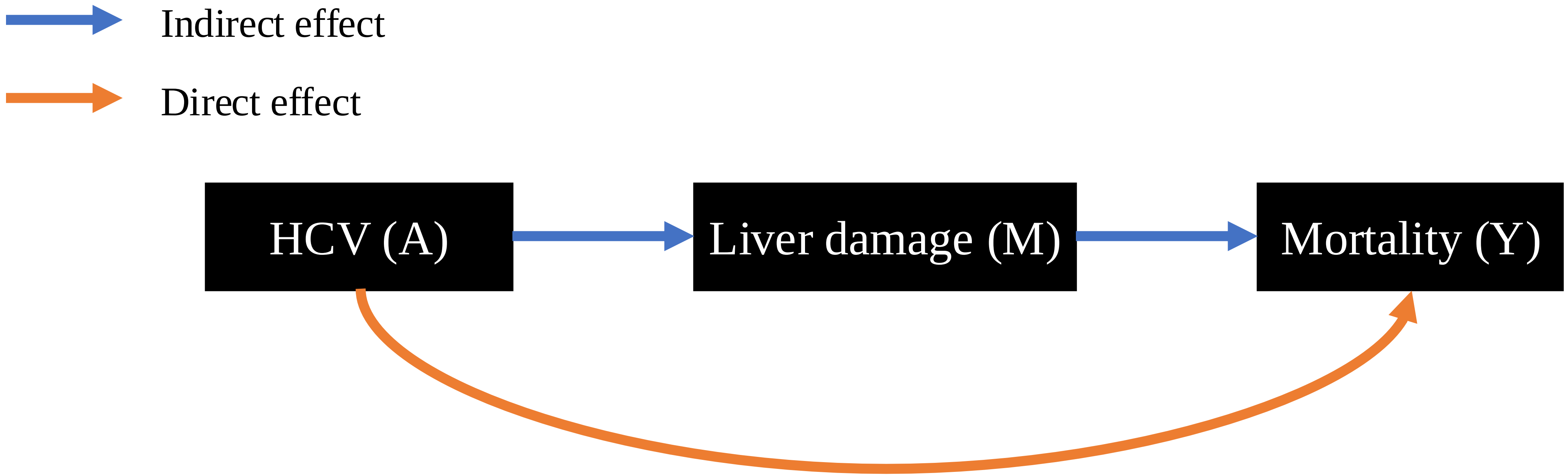
Descriptive interpretation (Natural direct and indirect effects)

- It is the effect of A if we let the mediator be whatever it naturally would have been under a particular scenario.

- Mechanism investigation

Direct and indirect effects

Effect decomposition of the total effect in the presence of a mediator



Prescriptive interpretation

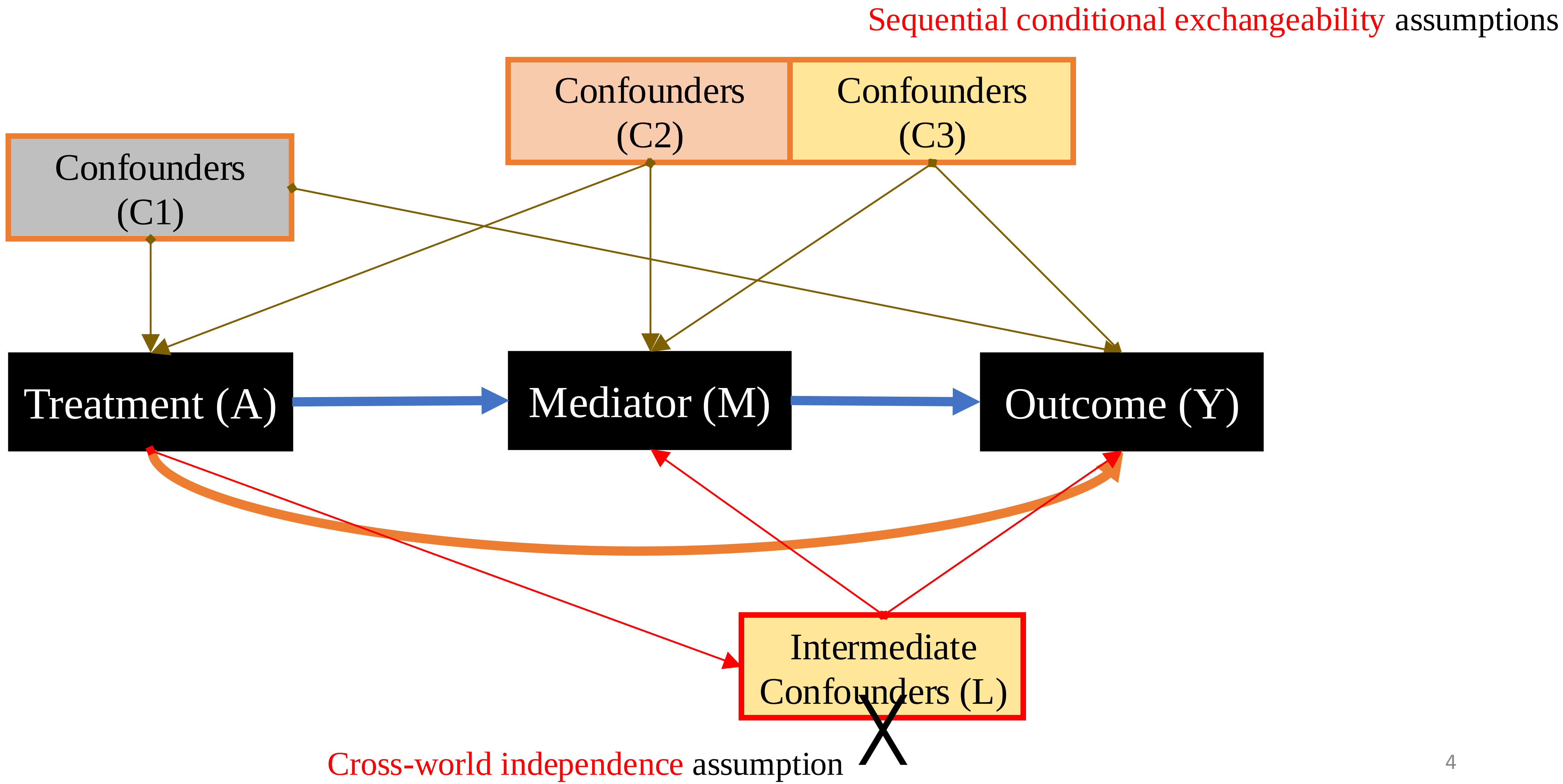
We may be interested in understanding what happens if a new public health policy is promoted and implemented.

Descriptive interpretation:

It aims to investigate the natural mechanism that HCV affects mortality in patients with liver disease.

Identification assumptions in mediation analysis

No unmeasured confounding



Cross-world independence assumption

Why are we not allowed to have an intermediated confounder?

Direct effect: $(A \rightarrow Y) + (A \rightarrow L \rightarrow Y)$
Indirect effect: $(A \rightarrow M \rightarrow Y) + (A \rightarrow L \rightarrow M \rightarrow Y)$

To Control or Not to Control?

- To control for the intermediated confounder

We would be blocking one of the direct effect pathways not through M that we were interested in.

- Not to control for the intermediated confounder

Direct and indirect effect estimates will be biased since L is a confounder of the M −Y relationship.

Cross-world independence assumption

A short conclusion for this assumption

Direct effect: $(A \to Y) + (A \to L \to Y)$
Indirect effect: $(A \to M \to Y) + (A \to L \to M \to Y)$

If an intermediated confounder is present in the mechanism, then…

1. The effects of $(A \to L \to Y)$ and $(A \to L \to M \to Y)$ cannot be distinguished empirically.

2. Natural direct and indirect effects are unidentifiable.

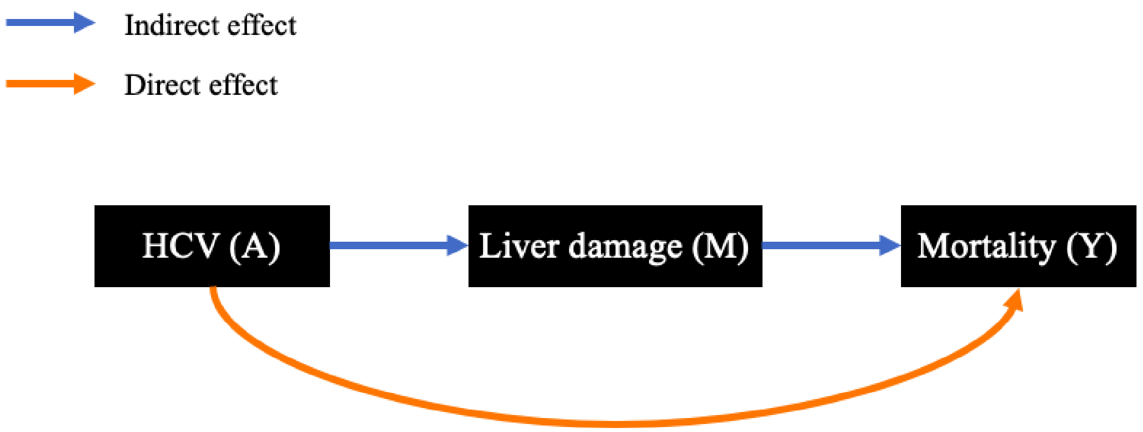
Identification and Estimation

Direct and indirect effects

For an individual, the counterfactuals are defined as

- Y(a,m): counterfactual outcome if the exposure and mediator had been a and m respectively.

- M(a'): counterfactual mediator if the exposure = a'.



- ➢ The effects of mediation analysis

Natural indirect effect (NIE): E(Y(1,M(1)))− E(Y(1,M(0)))

Natural direct effect (NDE): E(Y(1,M(0)))− E(Y(0,M(0)))

- ➢ Given identification assumptions of mediation analysis, E(Y(a,M(a′))) is identified as

$$\int_c \int_m E(Y|a, m, c_0)\, dF_{M|c}(m|a', c) dF_c(c).$$

Estimators of natural direct and indirect effects

- Given identification assumptions of mediation analysis, these are identified as

Natural direct effect

$$\int_c \left[\int_m E(Y|a=1,m,c)\, dF_{M|c}(m|a=0,c) - \int_m E(Y|a=0,m,c)\, dF_{M|c}(m|a=0,c) \right] dF_c(c)$$

Natural indirect effect

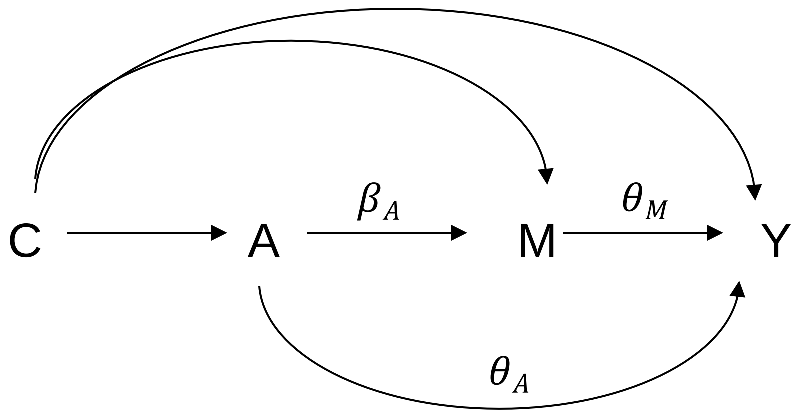
$$\int_c \left[\int_m E(Y|a=1,m,c)\, dF_{M|c}(m|a=1,c) - \int_m E(Y|a=1,m,c)\, dF_{M|c}(m|a=0,c) \right] dF_c(c)$$

- Estimation

- $E(Y|a,m,c)$
- $F_{M|c}(m|a,c)$
- $F_c(c)$

Product Method

- $E[Y|A,C] \;=\; \tau_0 \;+\; \tau_A A + \;\tau_C C$
- $E[Y|A,M,C] \;=\; \theta_0 \;+\; \theta_A A \;+\; \theta_M M + \;\theta_C C$
- $E[M|A,C] \;=\; \beta_0 \;+\; \beta_A A \;+\; \beta_C C$
- Total effect $= \tau_A$
- Natural Indirect Effect $\;=\; \beta_A \theta_M$



A summary for causal mediation analysis

- For causal inference, we should

  Research question → Causal structure → Assumptions → Identification → Statistical inference

- A series of exchangeability assumptions is crucial to avoid confounding.

1. No unmeasured A-Y confounding assumption (C1)
⟹ Verifiable, RCT

2. No unmeasured A-M confounding assumption (C2)
⟹ Verifiable, RCT

3. No unmeasured M-Y confounding assumption (C3)
⟹ It is theoretically verifiable but practically challenging.

4. No intermediated M-Y confounding assumption (L)
⟹ It cannot be tested and verified.