Causal inference HW3

1. The Lalonde dataset, introduced in Robert Lalonde's 1986 paper "Evaluating the Econometric Evaluations of Training Programs," is a classic example used to assess methods for estimating causal effects from observational data. It originates from the National Supported Work (NSW) Demonstration, a job training program conducted in the 1970s that provided employment opportunities to disadvantaged individuals. The primary objective is to estimate the causal effect of the job training program on post-treatment earnings in 1978, adjusting for confounding variables such as age, education, and pre-treatment income. Lalonde compared treatment effect estimates from the randomized experiment (NSW) to those obtained by combining the treated group with non-experimental comparison groups.

In this exercise, we will use a version of the dataset available in the **Matching** R package (note that this is not the original full dataset from Lalonde). This version includes 185 treated individuals from the NSW experimental group and 260 untreated individuals from a non-randomized comparison group.

Please load the dataset from the **Matching** package for the question below. Key variables used in this exercise are

Variable	Description
treat	1 if received treatment, 0 otherwise
age	Age at baseline
educ	Years of education
black	1 if Black
hispan	1 if Hispanic
married	1 if married
nodegree	1 if no high school diploma
re74	Real earnings in 1974 (pre-treatment)
re75	Real earnings in 1975 (pre-treatment)
re78	Real earnings in 1978 (post-treatment outcome)

Question:

Analyze the data using the following methods: (1) outcome regression, (2) matching based on Mahalanobis distance, (3) propensity score matching, (4) propensity score (PS) stratification, (5) regression with the PS as a covariate, (6) regression weighted by the inverse of the PS, (7) the Horvitz–Thompson estimator, (8) the Hájek estimator, (9) IPW with overlap weights, and (10) the doubly robust estimator. **Provide a comprehensive comparison of the results across all methods.**

2.

(A) Show that, under the assumption that the propensity score is known and exchangeability, the variance of the Hájek estimator is less than or equal to that of the Horvitz–Thompson estimator. That is, show that:

$$Var(\hat{\tau}_1^{HT}) \geq Var(\hat{\tau}^H)$$

(B) Trimming is commonly applied to IPW estimators to reduce the influence of extreme weights that arise when the estimated propensity scores are close to 0 or 1. Specifically, this involves removing observations whose estimated propensity scores $\hat{e}(X)$ fall outside the interval $[\alpha_L, \alpha_U]$. Conduct a simulation study to evaluate how different choices of α_L and α_U affect the performance of both the Horvitz–Thompson and Hájek estimators.